DEPENDENCE OF THE HEATING OF A HEAT-LIBERATING SURFACE ON THE HEAT-FLUX DENSITY IN EVAPORATION

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On the basis of numerical calculation of the conjugate problem of liquid boiling and heat conduction in a metallic wall, a functional dependence of the heating of a heat-liberating surface on the heat-flux density is obtained.

The influence of the heat-flux density q on the heating of a heat-liberating surface ΔT is usually approximated by a power law of the form

$$\Delta T = Cq^n . \quad (1)$$

In evaporation, the power index n may take different values in the range 1-0.05 [1-3]. Thus, for example, in boiling in a large volume, n in Eq. (1) varies from 0.8 at the onset of boiling to 0.3 in developed boiling. In the evaporation of liquid from some capillary structures of heat-pipe wicks, n is no greater than 0.05 [2]. It is natural to assume that the reason for this large spread in n in describing evaporation may be found in the boiling process itself, which is complex and multifactorial. Attempts to elucidate the factors responsible for the change in character of the dependence $\Delta T = \Delta T(q)$ in a direct physical experiment encounter considerable difficulties, since change in any one of the determining factors leads unavoidably to change in the other associated factors.

It is expedient to isolate the individual factors and analyze their appearance in a mathematical experiment on a model of the process.

In the numerical experiment with the results given below, the process of boiling (evaporation) of a layer heated to the water-saturation temperature on the metallic plate with the following sets of assumptions is analyzed:

1) the water layer is uniformly heated, and all the local heat sinks (centers of evaporation) are identical (in the physical experiment, this corresponds to evaporation from the surfaces of the meniscuses forming in pores of the capillary structure of the heat-pipe wicks);

2) the water layer is heated uniformly, but the appearance of a group of bubbles (or at least one bubble) influences the conditions of vapor-bubble appearance at other points of the surface at subsequent times (which corresponds to boiling in thin liquid films);

3) the water layer is nonuniformly heated, and its mixing depends on the quantity of bubbles forming (which corresponds to boiling in a large volume).

The following scheme of the process is considered. Above the free surface of the liquid, vapor at an unchanging pressure and saturation temperature T_s corresponding to this pressure is present. The temperature of the outer surface of the metallic wall is assumed to be constant in the course of a single calculation; the higher the temperature, the larger the heat flux which must be specified. The temperature of the wall surface immersed in the liquid T_L is determined from the equation of balance of the heat conducted from the wall by conduction and released into the liquid on account (in the general case) of conduction, evaporation, and mixing. The methods of calculation of this temperature depend on the assumption adopted and are considered in more detail below.

The heat conduction at the wall is described by a one-dimensional equation

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$$\frac{\partial T_{\text{M}}}{\partial \tau} = a \frac{\partial^2 T_{\text{M}}}{\partial x^2} . \tag{2}$$

The mass rate of water evaporation from the free surface is calculated from the formula of [4], which takes the following form after substitution of the physical constants:

$$V = 8.5754 \frac{\sqrt{T_{\text{S}}}}{v'} \left(1 - \frac{p}{p_{s\text{S}}}\right) \exp\left(-0.0021643 \frac{h'' - h'}{T_{\text{S}}}\right). \tag{3}$$

The vapor-phase formation in the liquid is described in accordance with the Zel'dovich–Kagan kinetic theory of nucleation [5].

The flux of vapor bubbles capable of further growth is determined by the equation

$$J = N_1 B \exp\left(-\frac{W}{k T_{\text{L}}}\right), \tag{4}$$

where the expression for the work of vapor-bubble formation of critical size takes the form

$$W = \psi \frac{16}{3} \frac{\pi\sigma^3}{(p_{s\text{L}} - p)^2 \left(1 - \frac{v'}{v''}\right)^2} . \tag{5}$$

It is assumed that the vapor bubbles grow in proportion to the square root of the time elapsed since their formation [6]

$$R = \frac{2\sqrt{3}}{\sqrt{\pi}} \frac{\lambda' v'' c}{r v'} (T_{\text{L}} - T_s) \sqrt{\tau_0} . \tag{6}$$

The maximum radius of the vapor bubble is bounded by the quantity R_{M}, corresponding to the breakdown dimension. After reaching its maximum dimension, the bubble ascends at a constant velocity corresponding to equality of the Archimedes and Stokes forces and breaks down on reaching the free surface.

Consider the balance of the heat reaching and leaving the liquid layer. The heat flux from the wall to the liquid by conduction is

$$Q_{\text{S}} = F\lambda_{\text{M}} \left.\frac{\partial T_{\text{M}}}{\partial x}\right|_{\text{L}} . \tag{7}$$

The heat flux leaving the surface Q_{O} is the sum of the heat fluxes required for evaporation of the liquid VF, kg/sec, from the free surface, formation of new vapor bubbles JFz, and growth of already-formed bubbles by the law in Eq. (6). Assuming that the temperature distribution over a liquid layer of thickness z is linear, the following expression may be written:

$$\frac{d(T_{\text{L}} + T_{\text{S}})}{d\tau} \frac{cFz}{2v'} = Q_{\text{S}} - Q_{\text{O}}. \tag{8}$$

Since the laws of vapor-bubble appearance used in the mathematical model are taken from the homogeneous theory of nucleation, strictly speaking, whereas the given case relates to the liquid–metal boundary, the theoretical heating is matched with the experimental values by selecting the corresponding value of the so-called form factor ψ in the expression for the work of bubble formation of critical size.

A series of calculations is performed for a massive copper plate of thickness 33.5 mm and area 10 cm^2; the thickness of the water layer is taken to be 1.8 mm and the pressure 0.1 MPa.

Identification of the model and experiment is based on the results of measuring the heat-transfer intensity on evaporation in thin liquid films [7, 8]. The bubble form factor ψ and its maximum radius R_{M} are selected so that the mean temperature difference at a heat-flux density of 40,000 W/m^2 is 8.34°K. It is assumed that $\psi = 0.47 \cdot 10^{-7}$ and $R_{\text{M}} = 2$ mm. The experimental point used in "tuning" the model is denoted by an asterisk in Fig. 1.

The first set of assumptions is realized with $T_{\text{S}} = T_{\text{L}}$ and ψ = const. The results of the calculations show that the mean temperature difference in boiling is practically independent of the thermal load in this case. Taking account of the error of the calculation, n does not exceed a value of the order of 0.001, i.e.,

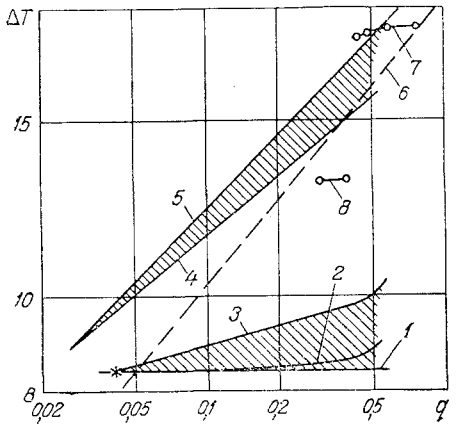

Fig. 1. Curves of bubble boiling: 1-3) with a uniformly heated liquid layer with various degrees of mutual influence of the evaporation centers (K_ψ = 1, 1.3, 1.7, respectively); 4, 5) with instantaneous mixing after bubble breakdown (K_ψ = 1 and 1.5, respectively); 6) bubble boiling of water in a large volume at a pressure 0.1 MPa according to the data of [10]; 7, 8) experimental data of [3] with boiling at different single centers in a large volume; the asterisk denotes the reference point taken for identification. ΔT, °K; q, MW/m².

$$\Delta T = Cq^{0.001}. \tag{9}$$

Curve 1 in Fig. 1 illustrates Eq. (9). Thus, a weak dependence of the heating in the course of boiling on the heat-flux density would be expected with an excess of identical evaporation centers (in comparison with the thermodynamically necessary number).

Such a weak dependence of the mean surface heating on q is observed with boiling on individual evaporation centers. Note, however, that experiments on boiling at individual centers [3] are conducted in conditions with a large liquid volume and characterized by larger ΔT than in the case of thin films.

The influence of one evaporation center on another at the maximum distribution density (at a distance of R_M) may be estimated considering the plate as a semiinfinite mass, knowing the power of the heat sink q_c represented by the growing vapor bubble and the contact time with the surface τ_M, from the following formula [9]:

$$\Delta T_L(R_M, \tau_M) = \frac{q_c}{2\pi\lambda_M R_M}\left[1 - \Phi\left(\frac{R_M}{\sqrt{4a\tau_M}}\right)\right]. \tag{10}$$

For a relatively small heat-flux density, the vapor bubble grows to its maximum size R_M = 2.5 mm in a time of the order of τ_M = 0.05 sec, while the heat-sink power observed at the end of the bubble lifetime is no greater than q_c = 10 W.

Calculations for the above numerical values of R_M, τ_M, and q_c for a copper plate show that this reduction in temperature is no more than 1.3°K, which corresponds to an increase in the work of bubble formation W by a factor of 1.3. The second assumption, that the appearance of a group of bubbles leads to a deterioration in the conditions for bubble formation, is realized by introducing a "relay" dependence of the bubble form factor ψ in Eq. (5) on the presence of bubbles. If there are no bubbles present, then $\psi = \psi_0$. If there is even one bubble, the work necessary for bubble formation will be K_ψ times larger, i.e., $\psi = \psi_0 K_\psi$.

The coefficient K_ψ varies over the range 1-1.7. The upper limit is chosen from calculations such that the whole possible range of influence of a single bubble on another is covered.

As would be expected, complication in the conditions of appearance of subsequent groups of bubbles is associated with increase in n. However, this increase is not unbounded. After reaching some value of K_ψ, the mean temperature difference ΔT and n cease to grow, since "limiting" deterioration occurs: the formation of only one group of bubbles, the first group, results in blocking of the formation of subsequent groups since the temperature difference required for the appearance of other bubbles in the worsened conditions is impossible within the lifetime of the first group. Hence, it becomes clear that cessation of the growth of ΔT and n with increase in K_ψ will set in earlier in processes with a smaller maximum radius R_M and at smaller heat-flux densities.

The whole range of theoretical boiling curves for a uniformly heated water layer with different degrees of influence of the bubbles appearing with the "neighbors" (from $K\psi = 1$ (curve 1 in Fig. 1) to $K_\psi = 1.7$ (curve 3 in Fig. 1)) lies in a narrow sector beneath the boiling curve for a large volume (curve 6, plotted from the data of [10]). Curve 3, corresponding to the case of the strongest mutual influence of the bubbles appearing, is described by an equation with n = 0.063:

$$\Delta T = Cq^{0.063}. \tag{11}$$

Hence it is evident that the mutual influence of evaporation centers is small. Even at the surface of the copper plate (high thermal conductivity), the operating evaporation center reduces the boundary temperature of its surrounding region R_M by no more than 1-3°K. In less heat-conducting materials, the mutual influence of the evaporation centers is even less.

Finally, the third assumption, that there is mixing in the liquid layer, corresponds, in our mathematical experiment, to variation in calculation of the temperature of the liquid–metal boundary.

It is assumed that the lower half of the liquid layer adjacent to the metal has a surface temperature of the metal T_L, and the temperature of the upper half of the liquid layer is the temperature of the free surface T_S. After bubble breakaway, an amount of liquid of temperature T_S and volume equal to the just-departed bubble takes its place at the liquid–metal boundary, and the same amount of liquid, at temperature T_L, reaches the surface. Heat conduction in the liquid is taken into account. As a result, the expression for the temperature of the immersed metal surface T_L may be written in finite-difference form as follows:

$$T_{L\tau+\Delta\tau} = T_{L\tau} + \frac{\left[Q_S - Q_0 + VFr - \lambda'(T_{L\tau} - T_S)\frac{2F}{z}\right]\Delta\tau - \frac{\chi z(T_{L\tau} - T_S)}{v'}}{\frac{czF}{2v'}}. \tag{12}$$

The influence of mixing alone (with $K_\psi = 1$) is shown in curve 4 in Fig. 1. Curve 5 shows the combined action of mixing and the mutual influence of the bubbles taken into account by the coefficient $K_\psi = 1.5$, which corresponds to cooling of the vicinity R_M of the evaporation center by approximately 2.6°K.

For curves 4 and 5, n = 0.206 and n = 0.236, respectively, in Eq. (1).

Thus, mixing of the liquid by ascending bubbles, especially in the case of large temperature gradients over the thickness of the liquid layer, leads to increase in the degree of dependence of ΔT on q. Note that the presence of a limited number of evaporation centers with different degrees of heating necessary for the initiation of bubble generation may also lead to increase in the degree of this dependence.

Although the three sets of assumptions discussed include simplifying assumptions — that the mixing is singular, that only two types of centers are present, with "easy" and "difficult" nucleation, and that there is no spatial distribution of evaporation centers — sufficiently general assumptions regarding the character of the boiling process — the presence or absence of interaction between the centers, evaporation, and mixing in the boiling-liquid layer — limit the maximum degree to which the heat-flux density influences the temperature difference in boiling and cover a fairly large range of experimental forms of the dependence $\Delta T = \Delta T(q)$.

NOTATION

τ, $\Delta\tau$, current time, theoretical time interval; τ_0, τ_M, time elapsed from bubble formation to the attainment of dimensions R and R_M, respectively; x, coordinate perpendicular to the heat-liberating surface; R, R_M, current and breakaway bubble radii; z, thickness of liquid layer; F, area of heat-liberating surface; χ, volume of the group of bubbles attained in the previous calculation step of the breakaway dimension; v', v", specific volumes of heated liquid and saturated vapor; V, mass evaporation rate from the free surface of the liquid; p, pressure above the water surface; p_{sL}, p_{sS}, saturation pressure corresponding to temperatures T_L, T_S; σ, surface tension; W, work of vapor-bubble formation; k, Boltzmann constant; ΔT, heating of the heat-liberating surface above the saturation temperature; T_S, T_L, temperatures of the free liquid surface and its boundary with the metal; T_M, temperature of the metallic wall; Q_S, Q_O, heat fluxes reaching and leaving the liquid; q, heat-flux density from the wall to the liquid; q_c, power of the heat sink represented by the growing vapor bubble; h', h", specific enthalpies of heated liquid and dry saturated vapor; r, effective heat of vaporization; c, specific heat of liquid; λ', λ_M, thermal conductivity of liquid and metal; a, thermal diffusivity of metal; ψ_0, ψ, initial and current form factor of bubble; B, preexponential factor depending on the liquid properties; N_1, number of molecules in unit volume of liquid; J, flux of stable new vapor bubbles; n, C, experimental constants; K_ψ, coefficient taking account of the increase in work of bubble formation on account of cooling of the surface of the adjacent growing bubble; Φ, Gaussian probability integral. Subscripts: L, metal—liquid boundary; τ, $\tau + \Delta\tau$, time interval currently being calculated and subsequent interval.

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